Maths

2n ESO

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Integers

1.1 Integers numbers and their representation

The set of integers numbers contains :

Naturals (counting) numbers 1, 2, 3, 4, ...

Zero, 0

Negative numbers -1, -2, -3, -4, ...

On the number line:

$$-5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$

1.2 Operations with integers

ADDITION

1) We add all positive numbers and it's positive +

2) We add all negative numbers and it's negative –

3) Find the absolute value of each integer. Subtract the smaller number from the larger number you get. The result takes the sign of the integer with the greater absolute value

Examples:

$a) 3-5-4+2+1-7+6=12-16=-4$

$b) -2-4-5=-11$

$c) 3+4+7+10=24$

In a)

We take positive numbers: 3, 2, 1, 6 and negative numbers: -5, -4, -7.

1) The sum of positive is 12
2) The sum of negative is -16.
3) The greatest absolute value is 16, so the result will be negative.

SUBSTRACTION

A negative in front of a set of parentheses changes the sign inside the parenthesis.

Exemple:

$$3-(-2)+5-(4-3)=3+2+5-4+3=13-4=9$$

-(-2) = 2
-(4-3) = -4+3

COMBINING ADDITION AND SUBSTRACTION

$$A)10-(13-7)=10-(6)=10-6=4$$
$$B)10-(13-7)=10-13+7=17-13=4$$

In A we simplify the operation inside the parentheses.
In B we substract both numbers in the parentheses.

Examples:

$$1)+(+8)-(-4)-(5)-(-3)+4=8+4-5+3+4=19-5=14$$
$$2)15-(12-8)=15-12+8=23-12=11$$
$$2)15-(12-8)=15-(4)=15-4=11$$
$$3)15-(6-9+5)=15-6+9-5=24-11=13$$
$$3)15-(6-9+5)=15-(11-9)=15-(2)=15-2=13$$
$$4)21-(3-10+11+6)=21-(20-10)=21-(10)=21-10=11$$
$$6)(5-16)-(7-3-6)-(9-13-5)=(-11)-(7-9)-(9-18)=-11-(-2)-(-9)=-11+2+9=0$$
$$7)6-[5+(8-2)]=6-[5+(6)]=6-(5+6)=6-(11)=6-11=-5$$
$$7)6-[5+(8-2)]=6-[5+8-2]=6-(13-2)=6-(11)=6-11=-5$$

MULTIPLYING

1) We multiply both numbers.
2) Use this rule to find the sign:

+·+ = +
+·- = -
-·+ = -
-·- = +

Examples:

$-2\cdot 3=-6\rightarrow -2\cdot 3=-2-2-2=-6$

$3\cdot(-6)=-18\rightarrow 3\cdot(-6)=-6-6-6=-18$

$(-3)\cdot(-5)\cdot 2=15\cdot 2=30$

$4\cdot 30=120$

DIVIDING

1) Dividie the numbers.
2) We use the same rule with the signs:

+:+ = +
+:- = -
-:+ = -
-:- = +

Examples:

$24:6=6\rightarrow 6\cdot 4=24$

$24:(-4)=-6$

$-24:4=-6$

$-24:(-6)=4$

<u>1.3 Combined operations with integers numbers</u>

L'ordre que hem de fer servir per a calcular operacions combinades és el següent:

1) First perform any calculations inside parentheses
2) Next perform all multiplications and divisions.
3) Lastly, perform all additions and subtractions.

Examples:

$-2\cdot(5-7)+4\cdot(-3)-2+1\cdot(-5+2)=$

$=-2\cdot(-2)+4\cdot(-3)-2+1\cdot(-3)=\rightarrow \textit{parentheses}$

$=4-12-2-3\rightarrow \textit{multiplications}$

$=4-17=-13$

Activities:

$a)-2\cdot(-3)+4-5\cdot(+2)-7=6+4-10-7=10-17=-7$

$b)4-3\cdot(-2)+6\cdot(-1+2)-4:(-2)=4-3\cdot(-2)+6\cdot(+1)-4:(-2)=$
$=4+6+6+2=18$

$c)15-8\cdot 3=15-24=-9$

$d)21-4\cdot 6+12:3=21-24+4=25-24=1$

$e)(-3)\cdot(-4)+(-6)\cdot 3=12-18=-6$

$f)18-5\cdot(3-8)=18-5\cdot(-5)=18+25=43$

$g)4\cdot(8-11)-6\cdot(7-9)=4\cdot(-3)-6\cdot(-2)=-12+12=0$

$h)(-2)\cdot[11+3\cdot(5-7)]-3\cdot(8-11)=$
$=(-2)\cdot[11+3\cdot(-2)]-3\cdot(-3)=$
$=(-2)\cdot(11-6)-3\cdot(-3)=$
$(-2)\cdot 5-3\cdot(-3)=-10+9=-1$

$i)28:(-7)-(-6)\cdot[23-5\cdot(9-4)]=$
$=28:(-7)-(-6)\cdot[23-5\cdot 5]=$
$=28:(-7)-(-6)\cdot(23-25)=28:(-7)-(-6)\cdot(-2)=-4-(+12)=-4-12=-16$

1.4 Powers of integers

If 'a' is an integer, the product of a x a x a x a x......xa for n times is denoted by a^n.

$$a^n=a\cdot a\cdot a\cdot a...\cdot a$$
$$a\rightarrow base$$
$$n\rightarrow \exp onent$$

Examples:

$$(+4)^2 = 4^2 = 4 \cdot 4 = 16$$
$$(-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) = 81$$
$$(-3)^5 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3) = -243$$

Powers of negative numbers

Raising a negative number to a power:

- **With an even exponent, it becomes positive**

$$(-8)^2 = (-8) \cdot (-8) = 64$$

- **With an odd exponent, it becomes negative**

$$(-8)^3 = (-8) \cdot (-8) \cdot (-8) = -512$$

Properties of powers

- **Power of a multiplication**

$$[(-2) \cdot 5]^3 = (-2)^3 \cdot 5^3$$
$$(-10)^3 = (-8) \cdot 125 \quad \rightarrow (a \cdot b)^n = a^n \cdot b^n$$
$$-1000 = -1000$$

- **Power of a division**

$$[(-10) : 5]^3 = (-10)^3 : 5^3$$
$$(-2)^3 = (-1000) : 125 \quad \rightarrow (a : b)^n = a^n : b^n$$
$$-8 = -8$$

- **Multiplying powers with the same base**

$$(-10)^2 \cdot (-10)^3 = (-10)^{2+3} = (-10)^5$$
$$100 \cdot (-1000) \qquad \rightarrow a^m \cdot a^n = a^{m+n}$$
$$-100000 = -100000$$

- **Dividing powers with the same base**

$$(-10)^5 : (-10)^3 = (-10)^{5-3} = (-10)^2$$
$$-100000 : (-1000) \qquad \rightarrow a^m : a^n = a^{m-n}$$
$$100 = 100$$

- **Power of a power**

$\left[(-10)^3\right]^2 = (-10)^{3\cdot 2} = (-10)^6$

$(-1000)^2$ $\rightarrow (a^n)^m = a^{n\cdot m}$

$1000000 = 1000000$

1.5 Square root of integers

A square root of a number a is a number b such that b2 = x, or in words, a number b whose square (the result of multiplying the number by itself) is a.

$$\sqrt{a} = b \Leftrightarrow b^2 = a$$

Numbers whose square roots are a whole number are called *perfect squares*.

Exemples:

$\sqrt{49} = 7 \Leftrightarrow 7^2 = 49$

$\sqrt{400} = 20 \Leftrightarrow 20^2 = 400$

49 and 400 are perfect squares

A positive number has two square roots

$$\sqrt{16} = \begin{cases} 4 \Leftrightarrow (-4)^2 = 16 \\ -4 \Leftrightarrow (-4)^2 = 16 \end{cases}$$

A negative number has no square root

$\sqrt{-16}$ doesn't exist. There is no number *a* that *a squared* is -16.

1.6 Other roots of integers

There are roots with an index greater than 2.

$$\sqrt[n]{a} = b \Leftrightarrow b^n = a$$

n is *the ´índex* and a is the *radicand*

Examples:

$$\sqrt[3]{8} = 2 \Leftrightarrow 2^3 = 8$$

$$\sqrt[4]{81} = \pm 3 \Leftrightarrow \begin{cases} 3^4 = 81 \\ (-3)^4 = 81 \end{cases}$$

$$\sqrt[3]{-8} = -2 \Leftrightarrow (-2)^3 = -8$$

$$\sqrt[4]{-81} \rightarrow No\_existeix$$

FRACTIONS

What is a Fraction?

A fraction is a number that expresses part of a group.

Fractions are written in the form $\frac{a}{b}$ or *a*/*b*, where a and b are whole numbers, and the number b is not 0. For the purposes of these web pages, we will denote fractions using the notation *a*/*b*, though the preferred notation is generally $\frac{a}{b}$.

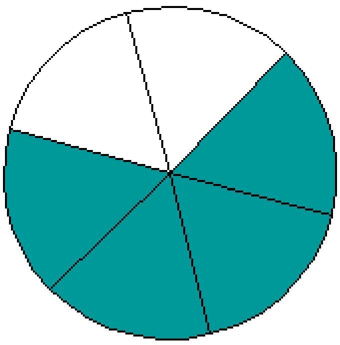
The number *a* is called the numerator, and the number *b* is called the denominator

Examples:

The following numbers are all fractions
1/2, 3/7, 6/10, 4/99

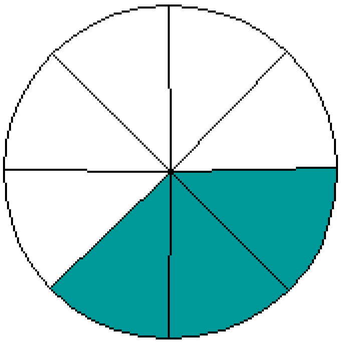
Example:

The fraction 4/6 represents the shaded portion of the circle below. There are 6 pieces in the group, and 4 of them are shaded.



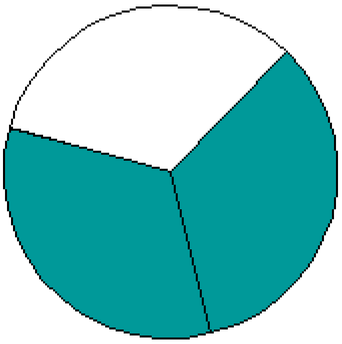
Example:

The fraction 3/8 represents the shaded portion of the circle below. There are 8 pieces in the group, and 3 of them are shaded.



Example:

The fraction 2/3 represents the shaded portion of the circle below. There are 3 pieces in the group, and 2 of them are shaded.



Equivalent Fractions

Equivalent fractions are different fractions which name the same amount.

Examples:

The fractions 1/2, 2/4, 3/6, 100/200, and 521/1042 are all equivalent fractions.
The fractions 3/7, 6/14, and 24/56 are all equivalent fractions.
We can test if two fractions are equivalent by cross-multiplying their numerators and denominators. This is also called taking the cross-product.

Example:

Test if 3/7 and 18/42 are equivalent fractions.
The first cross-product is the product of the first numerator and the second denominator: $3 \times 42 = 126$.
The second cross-product is the product of the second numerator and the

first denominator: 18 × 7 = 126.
Since the cross-products are the same, the fractions are equivalent.

Example:

Test if 2/4 and 13/20 are equivalent fractions.
The first cross-product is the product of the first numerator and the second denominator: 2 × 20 = 40.
The second cross-product is the product of the second numerator and the first denominator: 4 × 13 = 52.
Since the cross-products are different, the fractions are not equivalent.
Since the second cross-product is larger than the first, the second fraction is larger than the first.

Comparing Fractions

1. To compare fractions with the same denominator, look at their numerators. The larger fraction is the one with the larger numerator.
2. To compare fractions with different denominators, take the cross product. The first cross-product is the product of the first numerator and the second denominator. The second cross-product is the product of the second numerator and the first denominator. Compare the cross products using the following rules:
a. If the cross-products are equal, the fractions are equivalent.
b. If the first cross product is larger, the first fraction is larger.
c. If the second cross product is larger, the second fraction is larger.

Example:

Compare the fractions 3/7 and 1/2.
The first cross-product is the product of the first numerator and the second denominator: 3 × 2 = 6.
The second cross-product is the product of the second numerator and the first denominator: 7 × 1 = 7.
Since the second cross-product is larger, the second fraction is larger.

Example:

Compare the fractions 13/20 and 3/5.
The first cross-product is the product of the first numerator and the second denominator: 5 × 13 = 65.
The second cross-product is the product of the second numerator and the first denominator: 20 × 3 = 60.
Since the first cross-product is larger, the first fraction is larger.

Converting and Reducing Fractions

For any fraction, multiplying the numerator and denominator by the same nonzero number gives an equivalent fraction. We can convert one fraction to an equivalent fraction by using this method.

Examples:

1/2 = (1 × 3)/(2 × 3) = 3/6

2/3 = (2 × 2)/(3 × 2) = 4/6

3/5 = (3 × 4)/(5 × 4) = 12/20

Another method of converting one fraction to an equivalent fraction is by dividing the numerator and denominator by a common factor of the numerator and denominator.

Examples:

20/42 = (20 ÷ 2)/(42 ÷ 2) = 10/21

36/72 = (36 ÷ 3)/(72 ÷ 3) = 12/24

9/27 = (9 ÷ 3)/(27 ÷ 3) = 3/9

When we divide the numerator and denominator of a fraction by their greatest common factor, the resulting fraction is an equivalent fraction in lowest terms.

ALGEBRA

Algebraic Expressions

An *algebraic expression* is one or more algebraic terms in a phrase. It can include variables, constants, and operating symbols, such as plus and minus signs. It's only a phrase, not the whole sentence, so it doesn't include an equal sign.

Algebraic expression:

$$3x^2 + 2y + 7xy + 5$$

In an algebraic expression, terms are the elements separated by the plus or minus signs. This example has four terms, $\mathbf{3x^2}$, **2y**, **7xy**, and **5**. Terms may consist of variables and coefficients, or constants.

Variables

In algebraic expressions, letters represent variables. These letters are actually numbers in disguise. In this expression, the variables are x and y. We call these letters "**var**iables" because the numbers they represent can **vary**—that is, we can substitute one or more numbers for the letters in the expression.

Coefficients
Coefficients are the number part of the terms with variables. In **$3x^2 + 2y + 7xy + 5$**, the coefficient of the first term is 3. The coefficient of the second term is 2, and the coefficient of the third term is 7.

If a term consists of only variables, its coefficient is 1.

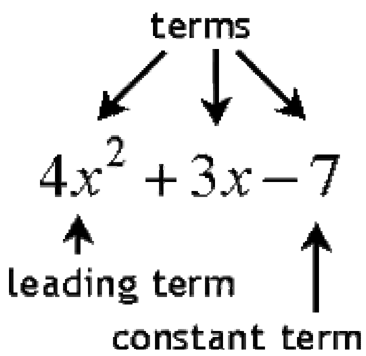
Constants
Constants are the terms in the algebraic expression that contain only numbers. That is, they're the terms without variables. We call them constants because their value never changes, since there are no variables in the term that can change its value. In the expression **$7x^2 + 3xy + 8$** the constant term is "8."

Polynomials: Definitions

By now, you should be familiar with variables and exponents. You may have dealt with expressions like $3x^4$ or $6x$. Polynomials are sums of these expressions. Each piece of the polynomial, each part that is being added, is called a "term". Polynomial terms have variables to whole-number exponents; there are no square roots of exponents, no fractional powers, and no variables in the denominator. Here are some examples:

$6x^{-2}$	NOT a polynomial term	This has a negative exponent.
$^1/_x2$	NOT a polynomial term	This has the variable in the denominator.
sqrt(x)	NOT a polynomial term	This has the variable inside a radical.
$4x^2$	a polynomial term	

Here is a typical polynomial:



Notice the exponents on the terms. The first term has exponent 2; the second term has an understood exponent 1; and the last term doesn't have any variable at all. Polynomials are usually written this way, with the terms written in "decreasing" order; that is, with the highest exponent first, the next highest next, and so forth, until you get down to the plain old number.

Any term that doesn't have a variable in it is called a "constant" term because, no matter what value you may put in for the variable *x*, that constant term will never change. In the picture above, no matter what*x* might be, 7 will always be just 7.

The first term in the polynomial, when it is written in decreasing order, is also the term with the biggest exponent, and is called the "leading term".

The exponent on a term tells you the "degree" of the term. For instance, the leading term in the above polynomial is a "second-degree term". The second term is a "first degree" term. The degree of the leading term tells you the degree of the whole polynomial; the polynomial above is a "second-degree polynomial". Here are a couple more examples:

- **Give the degree of the following polynomial: $2x^5 - 5x^3 - 10x + 9$**

  This polynomial has four terms, including a fifth-degree term, a third-degree term, a first-degree term, and a constant term.

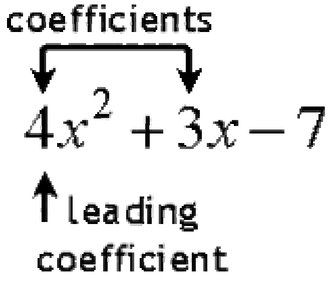
  This is a fifth-degree polynomial.

- **Give the degree of the following polynomial: $7x^4 + 6x^2 + x$**

  This polynomial has three terms, including a fourth-degree term, a second-degree term, and a first-degree term. There is no constant term.

  This is a fourth-degree polynomial.

When a term contains both a number and a variable part, the number part is called the "coefficient". The coefficient on the leading term is called the leading coefficient.



In the above example, the coefficient of the leading term is 4; the coefficient of the second term is 3; the constant term doesn't have a coefficient.

The "poly" in "polynomial" means "many". I suppose, technically, the term "polynomial" should only refer to sums of *many* terms, but the term is used to refer to anything from one term to a zillion terms. However, the shorter polynomials do have their own names:

- a one-term polynomial, such as $2x$ or $4x^2$, may also be called a "monomial"
- a two-term polynomial, such as $2x + y$ or $x^2 - 4$, may also be called a "binomial"
- a three-term polynomial, such as $2x + y + z$ or $x^4 + 4x^2 - 4$, may also be called a "trinomial"

I don't know if there are names for polynomials with a greater numbers of terms; I've never heard of any names other than what I've listed.

Polynomials are also sometimes named for their degree:

- a second-degree polynomial, such as $4x^{\mathbf{2}}$, $x^{\mathbf{2}} - 9$, or $ax^{\mathbf{2}} + bx + c$, is also called a "quadratic"
- a third-degree polynomial, such as $-6x^{\mathbf{3}}$ or $x^{\mathbf{3}} - 27$, is also called a "cubic"
- a fourth-degree polynomial, such as $x^{\mathbf{4}}$ or $2x^{\mathbf{4}} - 3x^2 + 9$, is sometimes called a "quartic"
- a fifth-degree polynomial, such as $2x^{\mathbf{5}}$ or $x^{\mathbf{5}} - 4x^3 - x + 7$, is sometimes called a "quintic"

There are names for some of the higher degrees, but I've never heard of any names being used other than the ones I've listed.

Evaluation

"Evaluating" a polynomial is the same as evaluating anything else: you plug in the given value of x, and figure out what y is supposed to be. For instance:

- **Evaluate $2x^3 - x^2 - 4x + 2$ at $x = -3$**

  Plug in −3 for x, remembering to be careful with parentheses and negatives:

  $$\begin{aligned} &2(-3)^3 - (-3)^2 - 4(-3) + 2 \\ &= 2(-27) - (9) + 12 + 2 \\ &= -54 - 9 + 14 \\ &= -63 + 14 \\ &= \mathbf{-49} \end{aligned}$$

Always remember to be careful with the minus signs!

Polynomials: Combining "Like Terms"

Probably the most common thing you will be doing with polynomials is "combining like terms". This is the process of adding together whatever terms you can, but not overdoing it by adding together terms that can't actually be combined. Terms can be combined if they have the exact same variable part. Here is a rundown of what's what:

$4x$ and 3	NOT like terms	The second term has no variable
$4x$ and $3y$	NOT like terms	The second term now has a variable, but it doesn't match the variable of the first term
$4x$ and $3x^2$	NOT like terms	The second term now has the same variable, but the degree is different
$4x$ and $3x$	LIKE TERMS	Now the variables match and the degrees match

Once you have determined that two terms are indeed "like" terms and can therefore be combined, you deal with them in a manner similar to what you did in grammar school. When you were first learning to add, you would do "five apples and six apples is eleven apples". You have since learned that, as they say, "you can't add apples and oranges". That is, "five apples and six oranges" is just a big pile of fruit; it isn't something like "eleven applanges". Combining like terms works much the same way. Here are some examples:

- **Simplify $3x + 4x$**

  These are like terms since they have the same variable part, so combine the terms: three x's and four x's makes seven x's:

  $$3x + 4x = 7x$$

- **Simplify $2x^2 + 3x - 4 - x^2 + x + 9$**

  It is often best to group like terms together first, and then simplify:

  $$2x^2 + 3x - 4 - x^2 + x + 9$$
  $$= (2x^2 - x^2) + (3x + x) + (-4 + 9)$$
  $$= x^2 + 4x + 5$$

In the second line, many students find it helpful to write in the understood coefficient of 1 in front of variable expressions with no written coefficient, as is shown in red below:

$(2x^2 - x^2) + (3x + x) + (-4 + 9)$
$= (2x^2 - \mathbf{1}x^2) + (3x + \mathbf{1}x) + (-4 + 9)$
$= 1x^2 + 4x + 5$
$= \boldsymbol{x^2 + 4x + 5}$

It is not required that the understood 1 be written in when simplifying expressions like this, but many students find this technique to be very helpful. Whatever method helps you consistently complete the simplification is the method you should use.

- **Simplify $10x^3 - 14x^2 + 3x - 4x^3 + 4x - 6$**

$10x^3 - 14x^2 + 3x - 4x^3 + 4x - 6$
$= (10x^3 - 4x^3) + (-14x^2) + (3x + 4x) - 6$
$= \boldsymbol{6x^3 - 14x^2 + 7x - 6}$

- **Simplify $25 - (x + 3 - x^2)$**

The first thing I need to do is take the negative through the parentheses:

$25 - (x + 3 - x^2)$
$= 25 - x - 3 + x^2$
$= x^2 - x + 25 - 3$
$= \boldsymbol{x^2 - x + 22}$

If it helps you to keep track of the negative sign, put the understood 1 in front of the parentheses:

$25 - (x + 3 - x^2)$
$= 25 - \mathbf{1}(x + 3 - x^2)$
$= 25 - 1x - 3 + 1x^2$
$= 1x^2 - 1x + 25 - 3$
$= 1x^2 - 1x + 22$
$\mathbf{=} \boldsymbol{x^2 - 1x + 22}$

While the first format (without the 1's being written in) is the more "standard" format, either format should be acceptable (check with your instructor). You should use the format that works most successfully for you.

- **Simplify $x + 2(x - [3x - 8] + 3)$**

This is the kind of problem that us math teachers love to put on tests (yes, we're cruel people), so you should *expect* to need to be able to do this. This is just an order of operations problem with a variable in it. Work carefully from the inside out, and you should be fine:

$x + 2(x - [3x - 8] + 3)$
$= x + 2(x - 1[3x - 8] + 3)$

$= x + 2(x - 3x + 8 + 3)$
$= x + 2(-2x + 11)$
$= x - 4x + 22$
$= \mathbf{-3x + 22}$

- **Simplify $[(6x - 8) - 2x] - [(12x - 7) - (4x - 5)]$**

  Work from the inside out:

  $[(6x - 8) - 2x] - [(12x - 7) - (4x - 5)]$
  $= [6x - 8 - 2x] - [12x - 7 - 4x + 5]$
  $= [4x - 8] - [8x - 2]$
  $= 4x - 8 - 8x + 2$
  $= \mathbf{-4x - 6}$

- **Simplify $-4y - [3x + (3y - 2x + \{2y - 7\}) - 4x + 5]$**

  $-4y - [3x + (3y - 2x + \{2y - 7\}) - 4x + 5]$
  $= -4y - [3x + (3y - 2x + 2y - 7) - 4x + 5]$
  $= -4y - [3x + (-2x + 5y - 7) - 4x + 5]$
  $= -4y - [3x - 2x + 5y - 7 - 4x + 5]$
  $= -4y - [3x - 2x - 4x + 5y - 7 + 5]$
  $= -4y - [-3x + 5y - 2]$
  $= -4y + 3x - 5y + 2$
  $= 3x - 4y - 5y + 2$
  $= \mathbf{3x - 9y + 2}$

$(x)(x) = x^2$ (multiplication)

$x + x = 2x$ (addition)

" x^2 " DOES NOT EQUAL " $2x$ "

So, if you have something like $x^3 + x^2$, DO NOT say that this somehow equals something like x^5 or $5x$. If you have something like $2x + x$, DO NOT say that this somehow equals something like $2x^2$.

Adding Polynomials

Adding polynomials is just a matter of combining like terms, with some order of operationsconsiderations thrown in. As long as you're careful with the minus signs, and don't confuse addition and multiplication, you should do fine.

There are a couple formats for adding and subtracting, and they hearken back to earlier times, when you were adding and subtracting just plain old numbers. First, you learned addition "horizontally", like this:
6 + 3 = 9. You can add polynomials in the same way, grouping like terms and then simplifying.

- **Simplify $(2x + 5y) + (3x - 2y)$**

  Clear the parentheses, group like terms, and simplify:

$$
\begin{aligned}
&(2x + 5y) + (3x - 2y) \\
&\quad = 2x + 5y + 3x - 2y \\
&\quad = 2x + 3x + 5y - 2y \\
&\quad = \mathbf{5x + 3y}
\end{aligned}
$$

Horizontal addition works fine for simple examples. But when you were adding plain old numbers, you didn't try to add 432 and 246 horizontally; instead, you would "stack" them vertically, one on top of the other, and add down:

$$
\begin{array}{r}
432 \\
+246 \\
\hline
678
\end{array}
$$

You can do the same thing with polynomials. This is how the above simplification looks when it is done "vertically":

- **Simplify $(2x + 5y) + (3x - 2y)$**

  I'll put each variable in its own column; in this case, the first column will be the *x*-column, and the second column will be the *y*-column:

$$
\begin{array}{rcr}
2x & + & 5y \\
3x & - & 2y \\
\hline
5x & + & 3y
\end{array}
$$

  I get the same solution vertically as I got horizontally: $\mathbf{5x + 3y}$.

The format you use, horizontal or vertical, is a matter of taste (unless the instructions explicitly tell you otherwise). Given a choice, you should use whichever format that you're more comfortable with. Note that, for simple additions, horizontal addition (so you don't have to rewrite the problem) is probably simplest, but, once the polynomials get complicated, vertical is probably safest (so you don't "drop", or lose, terms and minus signs). Here are some more examples:

- **Simplify $(3x^3 + 3x^2 - 4x + 5) + (x^3 - 2x^2 + x - 4)$**

  Horizontally:

$$
\begin{aligned}
&(3x^3 + 3x^2 - 4x + 5) + (x^3 - 2x^2 + x - 4) \\
&\quad = 3x^3 + 3x^2 - 4x + 5 + x^3 - 2x^2 + x - 4 \\
&\quad = 3x^3 + x^3 + 3x^2 - 2x^2 - 4x + x + 5 - 4 \\
&\quad = \mathbf{4x^3 + 1x^2 - 3x + 1}
\end{aligned}
$$

Vertically:

$$\begin{array}{r} 3x^3 + 3x^2 - 4x + 5 \\ \underline{x^3 - 2x^2 + x - 4} \\ 4x^3 + 1x^2 - 3x + 1 \end{array}$$

Either way, I get the same answer: $\mathbf{4x^3 + 1x^2 - 3x + 1}$.

Note that each column in the vertical addition contains only one degree of x: the first column is the x^3column, the second column is the x^2 column, the third column is the x column, and the fourth column is the constants column. This is analogous to having a thousands column, a hundreds column, a tens column, and a ones column.

- **Simplify $(7x^2 - x - 4) + (x^2 - 2x - 3) + (-2x^2 + 3x + 5)$**

It's perfectly okay to have to add three or more polynomials at once. I'll just go slowly and do each step throroughly, and it should work out right.

Horizontally:

$$\begin{aligned} &(7x^2 - x - 4) + (x^2 - 2x - 3) + (-2x^2 + 3x + 5) \\ &= 7x^2 - x - 4 + x^2 - 2x - 3 + -2x^2 + 3x + 5 \\ &= 7x^2 + \mathbf{1}x^2 - 2x^2 - \mathbf{1}x - 2x + 3x - 4 - 3 + 5 \\ &= 8x^2 - 2x^2 - 3x + 3x - 7 + 5 \\ &= \mathbf{6x^2 - 2} \end{aligned}$$

Note the 1's in the third line. Any time you have a variable without a coefficient, there is an "understood" 1 as the coefficient. If you find it helpful to write that 1 in, then do so.

Vertically:

$$\begin{array}{r} 7x^2 - x - 4 \\ x^2 - 2x - 3 \\ \underline{-2x^2 + 3x + 5} \\ 6x^2 \qquad - 2 \end{array}$$

Either way, I get the same answer: $\mathbf{6x^2 - 2}$

- **Simplify $(x^3 + 5x^2 - 2x) + (x^3 + 3x - 6) + (-2x^2 + x - 2)$**

Horizontally:

$$\begin{aligned} &(x^3 + 5x^2 - 2x) + (x^3 + 3x - 6) + (-2x^2 + x - 2) \\ &= x^3 + 5x^2 - 2x + x^3 + 3x - 6 + -2x^2 + x - 2 \\ &= x^3 + x^3 + 5x^2 - 2x^2 - 2x + 3x + x - 6 - 2 \\ &= \mathbf{2x^3 + 3x^2 + 2x - 8} \end{aligned}$$

When you add large numbers, there are sometimes zeroes in the numbers, such as:

$$\begin{array}{r} 1002 \\ 560 \\ \hline 1562 \end{array}$$

The zeroes in "1002" stand for "zero hundreds" and "zero tens". They are what is called "placeholders", indicating that there are no hundreds or tens. If you didn't have those zeroes there, you'd have "12", which isn't what you mean. The zeroes keep things lined up properly. When you vertically add polynomials that skip some of the degrees of *x*, you need to leave gaps, so the terms line up properly.

Vertically:

$$\begin{array}{rrrr} x^3 & + 5x^2 & - 2x & \\ x^3 & & + 3x & - 6 \\ & - 2x^2 & + x & - 2 \\ \hline 2x^3 & + 3x^2 & + 2x & - 8 \end{array}$$

Either way, you get the same answer: $\mathbf{2x^3 + 3x^2 + 2x - 8}$

Subtracting polynomials works pretty much the same way....

Subtracting Polynomials

Subtracting polynomials is quite similar to adding polynomials, but you have that pesky minus sign to deal with. Here are some examples, done both horizontally and vertically:

- **Simplify $(x^3 + 3x^2 + 5x - 4) - (3x^3 - 8x^2 - 5x + 6)$**

  The first thing I have to do is take that negative through the parentheses. Some students find it helpful to put a "1" in front of the parentheses, to help them keep track of the minus sign:

  Horizontally:

$$\begin{aligned} &(x^3 + 3x^2 + 5x - 4) - (3x^3 - 8x^2 - 5x + 6) \\ &\quad = (x^3 + 3x^2 + 5x - 4) - \mathbf{1}(3x^3 - 8x^2 - 5x + 6) \\ &\quad = x^3 + 3x^2 + 5x - 4 - 3x^3 + 8x^2 + 5x - 6 \\ &\quad = x^3 - 3x^3 + 3x^2 + 8x^2 + 5x + 5x - 4 - 6 \\ &\quad = \mathbf{-2x^3 + 11x^2 + 10x - 10} \end{aligned}$$

  Vertically:

$$\begin{array}{rrrrr} & x^3 & + 3x^2 & + 5x & - 4 \\ - & (3x^3 & - 8x^2 & - 5x & + 6) \\ \hline \end{array}$$

  In the horizontal case, you may have noticed that running the negative through the parentheses changed the sign on each term inside the parentheses. The shortcut here is to not bother writing in the subtaction sign or the parentheses; instead, I'll change all the signs in the second row (shown in red), and add down:

$$\begin{array}{rrrr} x^3 & +\ 3x^2 & +\ 5x & -\ 4 \\ -3x^3 & +\ 8x^2 & +\ 5x & -\ 6 \\ \hline -2x^3 & +\ 11x^2 & +\ 10x & -\ 10 \end{array}$$

Either way, I get the answer: $\mathbf{-2x^3 + 11x^2 + 10x - 10}$

- **Simplify $(6x^3 - 2x^2 + 8x) - (4x^3 - 11x + 10)$**

  Horizontally:

  $$\begin{aligned} &(6x^3 - 2x^2 + 8x) - (4x^3 - 11x + 10) \\ &= (6x^3 - 2x^2 + 8x) - 1(4x^3 - 11x + 10) \\ &= 6x^3 - 2x^2 + 8x - 4x^3 + 11x - 10 \\ &= 6x^3 - 4x^3 - 2x^2 + 8x + 11x - 10 \\ &= \mathbf{2x^3 - 2x^2 + 19x - 10} \end{aligned}$$

  Vertically:

  Write out the polynomials, leaving gaps as necessary:

  $$\begin{array}{rrrr} 6x^3 & -\ 2x^2 & +\ 8x & \\ 4x^3 & & -\ 11x & +\ 10 \\ \hline \end{array}$$

  ...and change the signs in the second line, and then add:

  $$\begin{array}{rrrr} 6x^3 & -\ 2x^2 & +\ 8x & \\ -4x^3 & & +\ 11x & -\ 10 \\ \hline 2x^3 & -\ 2x^2 & +\ 19x & -\ 10 \end{array}$$

  Either way, I get the answer: $\mathbf{2x^3 - 2x^2 + 19x - 10}$

Writing Algebraic Equations

Problem: Jeanne has \$17 in her piggy bank. How much money does she need to buy a game that costs \$68?

Solution: Let x represent the amount of money Jeanne needs. Then the following equation can represent this problem:

17 + x = 68

We can subtract 17 from both sides of the equation to find the value of x.

68 - 17 = x

Answer: x = 51, so Jeanne needs \$51 to buy the game.

In the problem above, x is a variable. The symbols **17 + x = 68** form an algebraic equation. Let's look at some examples of writing algebraic equations.

A number increased by nine is fifteen:	$y + 9 = 15$
Twice a number is eighteen	$2n = 18$
Four less than a number is twenty	$x - 4 = 20$
A number divided by six is eight	$\frac{k}{6} = 8$

Algebra Equations

Things you need to know to solve equations

Simple algebra equations are those that have only one variable.

Balance

The **most important** thing to remember when working with equations is to **always** do the **same thing** to **both sides of the equation**. This is called balance.

For example, if you add a number on the left side of the equation, then you also must add the same number to the right side of the equation.

$$x = 3$$
$$x + 2 = 3 + 2$$
$$x + 2 = 5$$

Keeping things in balance also works for subtraction, multiplication and division. Here is a multiplication example.

$$p = 5$$
$$2 \bullet p = 2 \bullet 5$$
$$2p = 10$$

Opposites

When solving equations in algebra, doing the **opposite** operation is usually required. One way to keep your equation **balanced** is to move things around by doing the opposite because you have to **undo** operations that have been done to the variable. The opposite of an operation is one way to move things around in an equation.

The opposite of adding three is subtracting three. If you add three to 100 you get 103. If you then subtract three from 103, you're back where you started. Adding 3 and subtracting 3 are opposites.

The opposite of subtraction is addition . The opposite of multiplication is division. The opposite of division is multiplication.

Like Terms

In algebra, a term is a part of an expression or equation that is separated from other parts by + or - signs. In the expression 3p + 7 there are two terms: 3p and 7. Here is a 5 term expression:

$$5a - 72 - 3a - 2xz + 8$$

In the expression above, notice that there are two terms that have the variable a, two terms with a number without a variable, and one term that has the variables x and z together. The 5a and the -3a terms are called "like terms" because they both have the variable a. They are alike! The number -72 and the number +8 are also like terms. The are alike because they are both pure numbers with no variable next to them. The -2xy term is different from all the other terms and cannot be combined with anything.

When you have like terms in algebra, they can be combined. If you put together the 5a and the -3a you get 2a because 5a - 3a = 2a. You can also put together the - 72 and the +8 terms and get -64 because - 72 + 8 = -64

Once you put together all the like terms, your expression is usually much simpler:

$$5a - 72 - 3a - 2xz + 8$$
$$2a - 64 - 2xz$$

Parentheses

In most equations (except those with fractions) you need to get rid of parentheses before you can do anything else. The way to do this is called the Distributive Property.

You know that in algebra when two things are next to each other it means that the two things are multiplied. For example 3e means 3 times e. This is also true when you have something next to a parentheses: 2(e + 3) means 2 times e **plus** 2 times 3. In algebra you would write it this way:

$$2(e + 3) = (2 \bullet e) + (2 \bullet 3) = 2e + 6$$

For most students, you can do the multiply step in the middle in your head, so your solution would look like this:

$$2(e + 3) = 2e + 6$$

You can also use the Distributive Property with subtraction:

$$5(e - 7) = 5e - 35$$

Fractions without Parentheses

Sometimes you need to solve an algebra equation that has fractions.

So remember

- numerator - the top part of a fraction
- denominator - the bottom part of a fraction

To get rid of fractions, first figure out the **common denominator**. The easiest way is to multiply all the denominators together - this might give you a bigger number than you really need, but it works if you are desperate.

Now multiply **everything on both sides of the equation** by the common denominator. The new equation will not have **any fractions - hurray!** Here is an example:

$\frac{1}{4}x + 3 + 2x = 5$	4 is the common denominator
$(4 \bullet \frac{1}{4}x) + (4 \bullet 3) + (4 \bullet 2x) = 4 \bullet 5$	multiply everything by 4
$x + 12 + 8x = 20$	simplify your answer
$9x + 12 = 20$	combine like terms - now you have a two-step equation
$9x + 12 = 20$ $-12 \quad -12$	subtract 12 from both sides of the equation
$\frac{9x}{9} = \frac{8}{9}$	divide both sides by 9
$x = \frac{8}{9}$	you are done! your answer has a fraction, but it wasn't hard to do

Fractions with Parentheses

When you have a fraction **and** parentheses the rules change a little bit. You still need the **common denominator**. If you forgot how, check the section on fractions without parentheses. Once the fraction is gone, then you will use the Distributive Property to help you get rid of any parentheses that are left.

This time we will **only** multiply the terms in parentheses and everything on the **other side** of the equation by the common denominator. Here is an example:

½(x + 5) = 6	2 is the common denominator
2 • ½(x + 5) = 2 • 6	multiply both sides by 2
x + 5 = 12	simplify your answer, a one-step equation

Steps for solving equations

Always try to solve equations in the order below. It might work if you use a different order, but it will add steps and make it harder. Most algebra problems only have some of the steps - not all those listed below.

1. First - get rid of any parentheses
2. Second - get rid of any fractions
3. Third - move all the variable terms by adding or subtracting (using the property of opposites) on both sides of the equal sign so the variable terms are all on one side of the equal sign
4. Fourth - move any number terms by adding or subtracting (using the property of opposites) on both sides of the equal sign so the number terms are all on the other side of the equal sign from the variable term
5. Fifth - combine like terms to get all the variable terms together and the number terms together
6. Sixth - if there is a number multiplied by the variable, get rid of it by dividing

Examples

One step problem - this only has the sixth step listed above because it is so easy.

3x = 15	Original Problem
3x ÷ 3 = 15 ÷ 3	to get rid of the 3 multiplied by the x, divide both sides of the equation by 3
x = 5	this is your answer

Two step problem - this requires you to do only the fifth and sixth steps listed above

2x - 4 = 8	Original Problem
2x - 4 + 4 = 8 + 4	move the -4 away from the 2x term by adding 4 to both sides of the equation

2x = 12	combine like terms, -4 + 4 = 0 and 8 + 4 = 12
2x ÷ 2 = 12 ÷ 2	to get rid of the 2 multiplied by the x, divide both sides of the equation by 2
x = 6	this is your answer

Example with fractions and no parentheses

¾x - 2 = 1	Original Problem
(4 • ¾x) - (4 • 2) = 4 • 1	get rid of the fraction by multiplying everything by the common denominator 4
3x - 8 = 4	do the math
3x - 8 + 8 = 4 + 8	move the -8 away from the 3x term by adding 8 to both sides of the equation
3x = 12	do the math
3x ÷ 3 = 12 ÷ 3	to get rid of the 3 multiplied by the x, divide both sides of the equation by 3
x = 4	this is your answer

Example with fractions and parentheses

¼(x + 2) = 8	Original Problem
4 • ¼(x + 2) = 4 • 8	get rid of the fraction by multiplying both sides by the common denominator 4
X + 2 = 32	do the math
x + 2 - 2 = 32 - 2	move the +2 away from the x term by subtracting 2 to both sides of the equation
x = 30	this is your answer

No Solution and Identity

You can't always get just one answer in algebra.

Sometimes when you are trying to solve a simple algebra problem you get a really weird looking answer. If the answer is a **big fat lie** like 5 = 3 or -7 = 2, we know that there is no answer to the problem. The algebra way to say big fat lie is **no solution**. As soon as you notice you have a big fat lie you get to stop and write down no solution for your answer. Here is an example:

2 (x + 4) = 3x - x + 5	Original Problem
2x + 8 = 3x - x + 5	distribute 2 times x + 4
2x + 8 = 2x + 5	combine like terms 3x - x
2x - 2x + 8 = 2x - 2x + 5	subtract 2x from both sides of the equation
8 = 5	combine the like terms 2x - 2x = 0
No solution	this is your answer because 8 does not equal 5 ever!

Another weird answer that sometimes happens is something that is **always true** like a = a or 6 = 6. This means that you can pick any answer at all to put back into the original equation and you will always get a true statement. The algebra way to say always true is**identity**. As soon as you notice you have something that is always true you get to stop and write down identity for your answer. Here is an example:

2 (a - 4) = 5a - 3a - 8	Original Problem
2a - 8 = 5a - 3a - 5	distribute 2 times x - 4
2a - 8 = 2a - 8	you get to stop now because you notice that you have the same stuff on **both sides** of your equal sign
Identity	this is your answer because 2a - 8 always equals itself!

MATHS WORD PROBLEMS

1. A father is 40 years old and his son is 10. How long will take until the father is twice as old as his son?
2. A rectangular estate is 150 m long. If it was 30 m longer and 20 m wider, its surface would be 6000 m^2 larger. Which are the dimensions of the estate?
3. If we mix up 2€/l wine and 3,5€/l wine we get 500 litres of medium quality wine that costs 2,90 €/l. How many litres of each wine have we needed?
4. A pedestrian and a cyclist are on the same road in opposite directions. Their speed are 6 km/h and 24 km/h. How long will it take to meet if they are 8 km apart? Which distance will the pedestrian walk?

5. I have bought 5 ballpens and I have 2 euros left. If I had bought 9 ballpens, I would have needed one more euro. How much is each ballpen? How much did I have?

6. A tank has got two taps, A and B. If we only turn A on, the tank gets filled in 3 hours. If we turn both on, it gets filled in 2 hours. How long will it take to fill the tank only from the tap B?

7. I have spent one-fifth of my pay in a comic and one-fourth with my friends. Now I'm catching the bus, that is 1,10 € and I have 4,40 euros left. How much is my pay?

8. Find out three consecutive even numbers if we know that the first one doubled plus the third one equals the second one plus 10.

9. Find out the angles of a right triangle if one of them is 30 degrees lower than the other.

10. Alice is twice Joe's age, and Jack is three years older than Alice. The sum of their age is 38, how old is every one?

11. How many litres of sunflower oil, 1,5 €/l, must we mix up with 15 litres of olive oil 7,5 €/l if we want a mixture of 6€/l?

12. One pipe can fill a pool in 14 hours. Another pipe can do it in 16 hours. How long would I take both together?

13. In reductions I have bought three shirts and two trousers and I have paid 17,5 €. I can remember that the price of one trousers doubles the price of one shirt. How much was every thing?

14. One truck leaves a town at 80 km per hour. Forty five minutes later, a car goes in its pursuit at 120 km per hour. How long will it take to meet it?

15. The perimeter of an equilateral triangle increased in half one side's length and then multiplyed by two, we get 28. How long is the side?

16. A father is 43 years old and his sons 13 and 10. When will the age of the bothers together amount the same the age of the father?

17. A worker does something in 3 hours. His assistant needs 12 hours to do the same thing. What it would be if they do it together?

18. There are 200 litres of water in a vessel after taking out its half and then 1/3 of what remained in it. How many litres were there?

19. John and his brother's ages are half their father's age. If John is 14 and his father is 6 times his brither's age, how old is John's brother?

20. A train from Sevilla to Madrid leaves at 240 km per tour. At the same time another train leaves from Madrid to Sevilla at 60 km per hour. Sevilla and Madrid are 542 km apart. How long will it take till they meet? How far from Madrid will it be?

21. Jane has travelled to Tenerife and wants to hire a car for four days. She reads this ad: Rent a car. Choose your fare. A: 55 € per day and

0,47 € per km or B. 139,60 per day. Write down the algebraic expressions to calculate the cost. Which fare is better?

22. Angela is 28 years younger than her father and 24 years older than her son. How old is everyone if their ages amounts 100 years.

23. The sides of a isosceles trapezium measure 8 cm, one base doubles the other. Find out their lengths if the perimeter is 46 cm.

24. A reaping machine needs 2 hours to cut the grass. A man would spend 28 hours to do the same. How long would it take to do the work, both together?
25. If we mixed up one oil, 8 €/l with another oil of 6 €/l we get 20 liters of medium quality oil, 6,70 €/l. How many liters of each have we used?
26. Calculate the amount of money that three people have shared 3000 € out if the first one gets 65 € more than the second one and this, 200 € more than the third one.
27. My grandfather is 69 years old. If I was 15 years older than I am and he was 15 years younger than he is, my age will be half his age. How old am I?
28. Find the length and the width of a rectangular estate if its length is twice the width plus and they have used 850 metres of fence.
29. How much did Marta carry if after being out with her friends and spending 3/5 of her money she has 8,58 left?
30. Carles has got certain amount of money and Nuria has got twice. Joan has got half Carles' amount and Raquel has got 1/3 of what Nuria has. Everything put together is 85 €. How much has evey one got?
31. They have mixed 10 litres of good milk with 8 litres of not so good milk. So they have obtained a medium quality milk at 1,10 €/l. We know that the good milk is 0,36 € more expensive than the other one. How much is every milk?
32. A car and a truck leave at the same time two towns, A and B, which are 380 km apart. They cross at a point 60 km nearer from B than from A. The speed of the truck is 80 km per hour. How long will it take to meet? How fast does the car go?
33. A pipe fills a pool in 8 hours, another pipe in 3 hours and a drain empties it in 4 hours. How long would it take to fill the pool with the two pipes and the drain on?
34. In a rectangle, the base is twice the height. If the base would be 3 cm shorter and the height 4 cm longer, we would have a new rectangle with 32 cm of perimeter. What dimensions has the first rectangle got?
35. Joan is 4 years older than his sister and 6 years before he was twice as old as her sister. How old are they?

QUADRATIC EQUATIONS

A quadratic equation is an equation that has a second-degree term and no higher terms. A second-degree term is a variable raised to the second power, like x^2.

A quadratic equation can be written in the form:

$$ax^2 + bx + c = 0$$

,

where *a*, *b*, and *c* are numbers (*a* ≠0), and x is the variable. x is a solution (or a root) if it satisfies the equation $ax^2 + bx + c = 0$.

Solving a Quadratic Formula:

Some quadratic equations can be solved easily by factoring.

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \sqrt{1}$$

$$x = \pm 1$$

or

$$x^2 - 3x = 0$$

$$x\,(x - 3) = 0$$

$$x = 0 \text{ and } x - 3 = 0 \text{ that is } x = 3$$

Most second-degree equations are more difficult to solve, and cannot be solved by simple factoring. The quadratic formula is a general way of solving any quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula gives two solutions, one double solution or no solution.

SYSTEMS OF EQUATIONS

To find the solution to systems of linear equations, you can use three methods

- algebraic equation : by setting the equations of the system equal to each other then solving this equation.
- substitution : by solving for one of the variables and substituting its value in to the other equation.
- Elimination : Elimination involves algebraic manipulations of two or more equations. The end goal is to eliminate a variable by creating opposite coefficients

SUBSTITUTION

The method of solving "by substitution" works by solving one of the equations (you choose which one) for one of the variables (you choose which one), and then plugging this back into the other equation, "substituting" for the chosen variable and solving for the other. Then you back-solve for the first variable. Here is how it works.

Solve the following system by substitution.

$$\mathbf{2x - 3y = -2}$$
$$\mathbf{4x + y = 24}$$

The idea here is to solve one of the equations for one of the variables, and plug this into the other equation. It does not matter which equation or which variable you pick. There is no right or wrong choice; the answer will be the same, regardless. But — some choices may be better than others.

For instance, in this case, can you see that it would probably be simplest to solve the second equation for "$y =$", since there is already a y floating around loose in the middle there? I could solve the first equation for either variable, but I'd get fractions, and solving the second equation for x would also give me fractions. It wouldn't be "wrong" to make a different choice, but it would probably be more difficult. Solve the second equation for y:

$$4x + y = 24$$
$$y = -4x + 24$$

Now I'll plug this in ("substitute it") for "y" in the first equation, and solve for x:

$$2x - 3(-4x + 24) = -2$$
$$2x + 12x - 72 = -2$$

$14x = 70$
$x = 5$

Now I can plug this x-value back into either equation, and solve for y. But since I already have an expression for "$y =$", it will be simplest to just plug into this:

$y = -4(5) + 24 = -20 + 24 = 4$

Then the solution is $(x, y) = (5, 4)$.

ELIMINATION

The addition method of solving systems of equations is also called the method of elimination. This method is similar to the method you probably learned for solving simple equations. If you had the equation "$x + 6 = 11$", you would write "-6" under either side of the equation, and then you'd "add down" to get "$x = 5$" as the solution.

$$\begin{array}{rcr} x + 6 & = & 11 \\ \underline{\;\;-6} & & \underline{-6} \\ x \;\;\;\;\; & = & 5 \end{array}$$

You'll do something similar with the addition method.

- **Solve the following system using addition.**

  $2x + y = 9$
  $3x - y = 16$

  Note that, if I add down, the y's will cancel out. So I'll draw an "equals" bar under the system, and add down:

$$\begin{array}{l} 2x + y = 9 \\ \underline{3x - y = 16} \\ 5x \quad\;\; = 25 \end{array}$$

  Now I can divide through to solve for $x = 5$, and then back-solve, using either of the original equations, to find the value of y. The first equation has smaller numbers, so I'll back-solve in that one:

$$\begin{array}{r} 2(5) + y = 9 \\ 10 + y = 9 \\ y = -1 \end{array}$$

  Then the solution is $(x, y) = (5, -1)$.

It doesn't matter which equation you use for the backsolving; you'll get the same answer either way. If I'd used the second equation, I'd have gotten:

$$3(5) - y = 16$$
$$15 - y = 16$$
$$-y = 1$$
$$y = -1$$

...which is the same result as before.

The Algebraic Equation Method

Let's take another look at the system of equations from above:

- y=2x+1
- y=4x-1

By examining the graph we can see that the point of intersection, or the solution, is the point (1,3) where the lines intersected.

Steps for the algebraic method:

Make sure that each linear equation is reduced to slope intercept form

(ie y=3x+2 is good but 2y=6x+4 is NOT)

Set the two equations equal to each other

$$2x+1=4x-1$$

Solve for X

$$2x+1=4x-1$$
$$2=2x$$
$$x= 1$$

Insert x value into either equation to determine y coordinate of solution
4(1)-1=3

The solution is the ordered pair you've just calculated

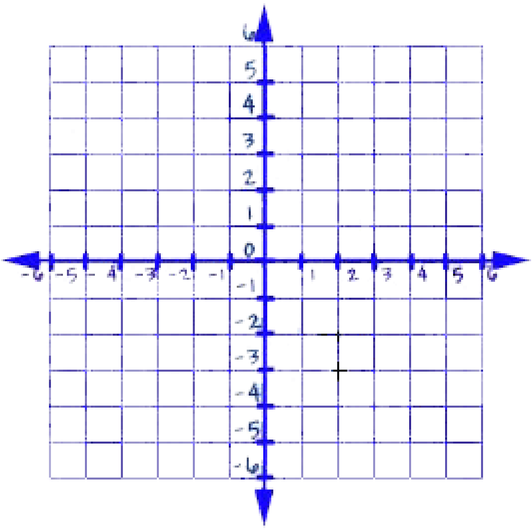
1. How many rabbits and hens were there in a farm if Llúcia got 61 heads and Ricard 196 legs.
2. We mix up oil 8 €/l and oil 6€/l and we obtain 20 litres of medium quality oil at 6,7€/l. How many litres of each oil do we need?
3. Dolors is six times as old as her grandson Carles, but in eights years time she will be only four times. How old are every one?

4. Miquel’s wage is three times his son’s wage. Enric, his son, is going to earn 400 more euros next month and that will be half his father’s wage. How much do they earn now?
5. We mix up 2€/l and 3,5€/l wine and obtain 500 litres of medium quality at 2,9€/l. How many litres of each do we need?
6. There are 20 coins in my pocket, 20 cents and 50 cents. In total I have got 7,90 euros. How many coins of each do I have?
7. Which amount of gold 8 €/g and silver 1,70 €/g do we need for one kilo of mixture 4,22 €/g?

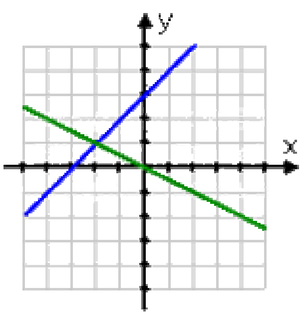
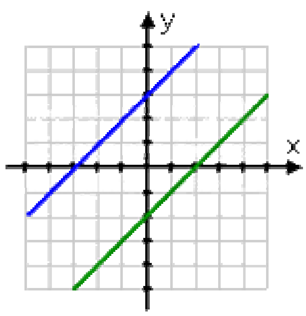
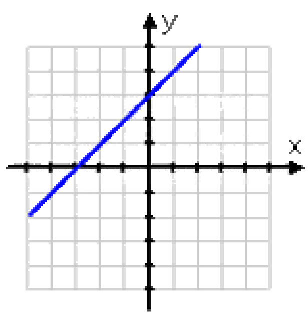
The coordinate plane

A coordinate plane is an important tool for working with these equations. It is formed by a horizontal number line, called the x-axis, and a vertical number line, called the y-axis. The two axes intersect at a point called the origin.

You can locate any point on the coordinate plane by an ordered pair of numbers (x,y), called the coordinates.



When you are solving systems, you are, graphically, finding intersections of lines. For two-variable systems, there are then three possible types of solutions:

Case 1	Case 2	Case 3
		

The first graph above, "Case 1", shows two distinct non-parallel lines that cross at exactly one point. This is called an "independent" system of equations, and the solution is always some x,y-point.

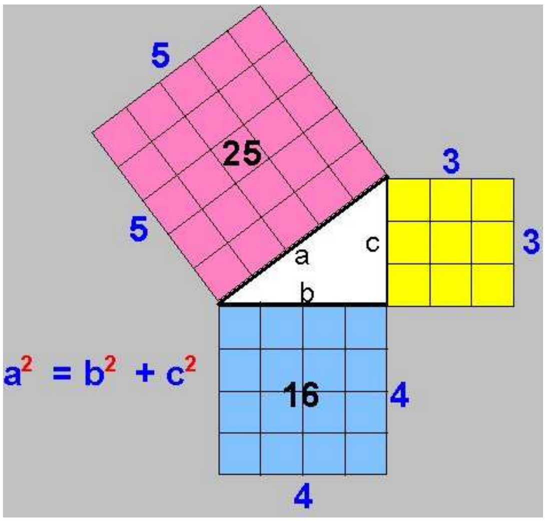
The second graph above, "Case 2", shows two distinct lines that are parallel. Since parallel lines never cross, then there can be no intersection; that is, for a system of equations that graphs as parallel lines, there can be no solution. This is called an "inconsistent" system of equations, and it has no solution.

The third graph above, "Case 3", appears to show only one line. Actually, it's the same line drawn twice. These "two" lines, really being the same line, "intersect" at every point along their length. This is called a "dependent" system, and the "solution" is the whole line.

This shows that a system of equations may have one solution (a specific x,y-point), no solution at all, or an infinite solution (being all the solutions to the equation).

Geometry

First of all, let's remember Pythagoras' theorem



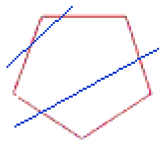
What is a Polygon?

A closed plane figure made up of several line segments that are joined together. The sides do not cross each other. Exactly two sides meet at every vertex.

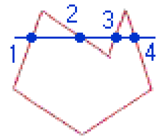
Types of Polygons

Regular - all angles are equal and all sides are the same length. Regular polygons are both equiangular and equilateral.

Equiangular - all angles are equal.
Equilateral - all sides are the same length.

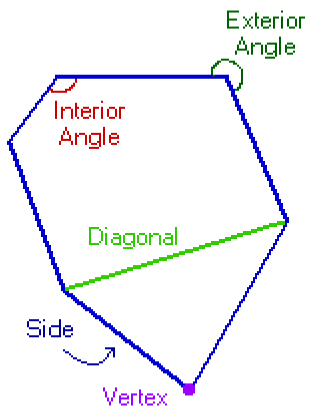


Convex - a straight line drawn through a convex polygon **crosses at most two sides**. Every interior angle is less than 180°.



Concave - you can draw at least one straight line through a concave polygon that **crosses more than two sides**. At least one interior angle is more than 180°.

Polygon Parts



Side - one of the line segments that make up the polygon.

Vertex - point where two sides meet. Two or more of these points are called vertices.

Diagonal - a line connecting two vertices that isn't a side.

Interior Angle - Angle formed by two adjacent sides inside the polygon.

Exterior Angle - Angle formed by two adjacent sides outside the polygon.

Special Polygons

Special Quadrilaterals - square, rhombus, parallelogram, rectangle, and the trapezoid.

Special Triangles - right, equilateral, isosceles, scalene, acute, obtuse.

Polygon Names

Generally accepted names

Sides	Name
n	N-gon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon

10	Decagon
12	Dodecagon

Area

The area of a figure measures the size of the region enclosed by the figure. This is usually expressed in terms of some square unit. A few examples of the units used are square meters, square centimeters, square inches, or square kilometers.

Area of a Square

If *l* is the side-length of a square, the area of the square is l^2 *or* $l \times l$.

Example:

What is the area of a square having side-length 3.4?
The area is the square of the side-length, which is 3.4 × 3.4 = 11.56.

Area of a Rectangle

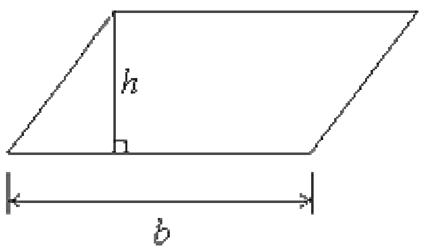
The area of a rectangle is the product of its width and length.

Example:

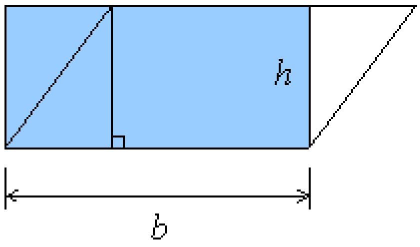
What is the area of a rectangle having a length of 6 and a width of 2.2?
The area is the product of these two side-lengths, which is 6 × 2.2 = 13.2.

Area of a Parallelogram

The area of a parallelogram is $b \times h$, where *b* is the length of the base of the parallelogram, and *h* is the corresponding height. To picture this, consider the parallelogram below:



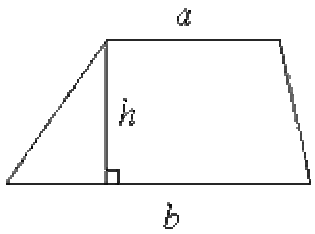
We can picture "cutting off" a triangle from one side and "pasting" it onto the other side to form a rectangle with side-lengths *b* and *h*. This rectangle has area *b* × *h*.



Example:

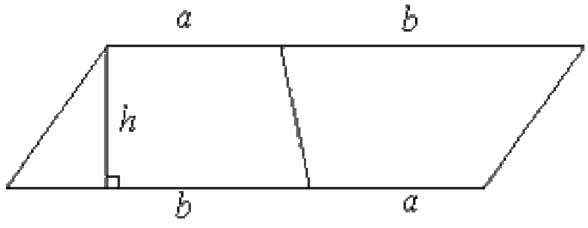
What is the area of a parallelogram having a base of 20 and a corresponding height of 7?
The area is the product of a base and its corresponding height, which is 20 × 7 = 140.

Area of a Trapezoid



If *a* and *b* are the lengths of the two parallel bases of a trapezoid, and *h* is its height, the area of the trapezoid is
1/2 × *h* × (*a* + *b*) .

To picture this, consider two identical trapezoids, and "turn" one around and "paste" it to the other along one side as pictured below:

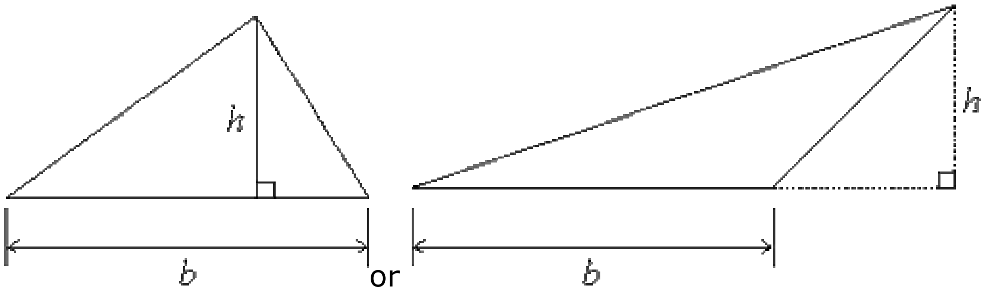


The figure formed is a parallelogram having an area of *h* × (*a* + *b*), which is twice the area of one of the trapezoids.

Example:

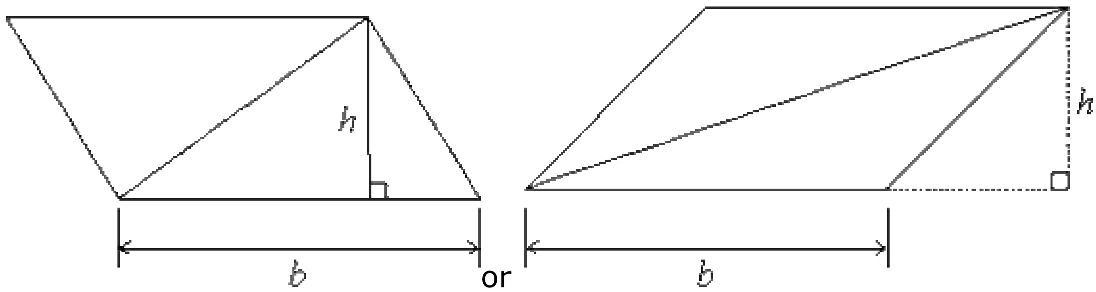
What is the area of a trapezoid having bases 12 and 8 and a height of 5?
Using the formula for the area of a trapezoid, we see that the area is
1/2 × 5 × (12 + 8) = 1/2 × 5 × 20 = 1/2 × 100 = 50.

Area of a Triangle



Consider a triangle with base length *b* and height *h*.
The area of the triangle is 1/2 × *b* × *h*.

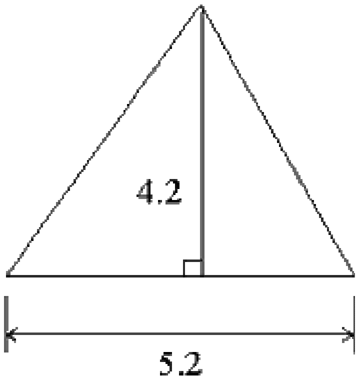
To picture this, we could take a second triangle identical to the first, then rotate it and "paste" it to the first triangle as pictured below:



The figure formed is a parallelogram with base length *b* and height *h*, and has area *b* × ×*h*.
This area is twice that of the triangle, so the triangle has area 1/2 × *b* × *h*.

Example:

What is the area of the triangle below having a base of length 5.2 and a height of 4.2?
The area of a triangle is half the product of its base and height, which is 1/2 ×5.2 × 4.2 = 2.6 × 4.2 = 10.92..



Area of a Circle

The area of a circle is Pi $\times r^2$ or Pi $\times r \times r$, where r is the length of its radius. Pi is a number that is approximately 3.14159.

Example:

What is the area of a circle having a radius of 4.2 cm, to the nearest tenth of a square cm? Using an approximation of 3.14159 for Pi, and the fact that the area of a circle is Pi $\times r^2$, the area of this circle is Pi $\times 4.2^2 \cong 3.14159 \times 4.2^2$ =55.41…square cm, which is 55.4 square cm when rounded to the nearest tenth.

Perimeter

The perimeter of a polygon is the sum of the lengths of all its sides.

Example:

What is the perimeter of a rectangle having side-lengths of 3.4 cm and 8.2 cm? Since a rectangle has 4 sides, and the opposite sides of a rectangle have the same length, a rectangle has 2 sides of length 3.4 cm, and 2 sides of length 8.2 cm. The sum of the lengths of all the sides of the rectangle is 3.4 + 3.4 + 8.2 + 8.2 = 23.2 cm.

Example:

What is the perimeter of a square having side-length 74 cm? Since a square has 4 sides of equal length, the perimeter of the square is
74 + 74 + 74 + 74 = 4 × 74 = 296.

Example:

What is the perimeter of a regular hexagon having side-length 2.5 m? A hexagon is a figure having 6 sides, and since this is a regular hexagon, each side has the same length, so the perimeter of the hexagon is
2.5 + 2.5 + 2.5 + 2.5 + 2.5 + 2.5 = 6 × 2.5 = 15m.

Example:

What is the perimeter of a trapezoid having side-lengths 10 cm, 7 cm, 6 cm, and 7 cm? The perimeter is the sum 10 + 7 + 6 + 7 = 30cm.

Circumference of a Circle

The distance around a circle. It is equal to Pi (π) times the diameter of the circle. Pi or π is a number that is approximately 3.14159.

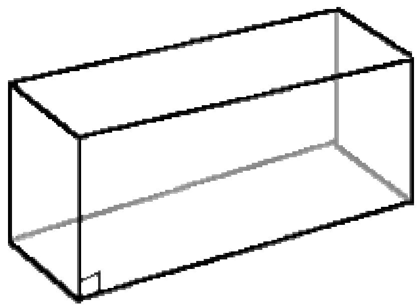
Example:

What is the circumference of a circle having a diameter of 7.9 cm, to the nearest tenth of a cm? Using an approximation of 3.14159 for π, and the fact that the circumference of a circle is π times the diameter of the circle, the circumference of the circle is Pi × 7.9 $\cong$ 3.14159 × 7.9 = 24.81…cm, which equals 24.8 cm when rounded to the nearest tenth of a cm.

Area and Volume of Solids

Area of Prisms

There are special formulas that deal with prisms, but they only deal with right prisms. *Right prisms* are prisms that have two special characteristics - all lateral edges are perpendicular to the bases, and lateral faces are rectangular. The figure below depicts a right prism.

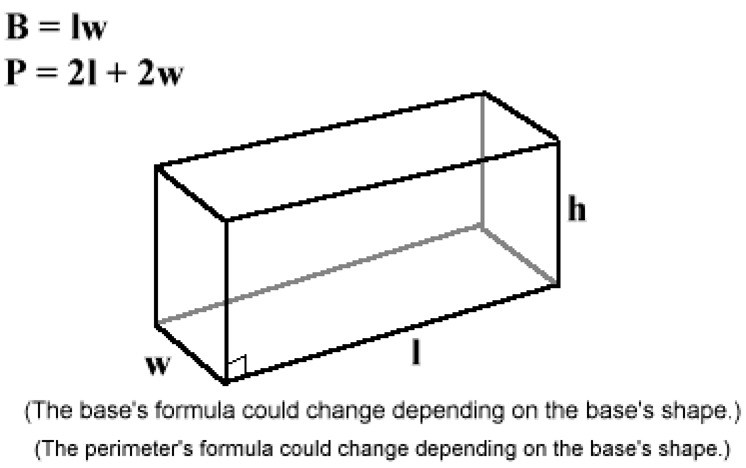


<u>Right Prism Area</u>

The lateral area *L* (area of the vertical sides only) of any right prism is equal to the perimeter of the base times the height of the prism (*L = Ph*).

The total area *T* of any right prism is equal to two times the area of the base plus the lateral area.

Formula: *T = 2B + Ph*

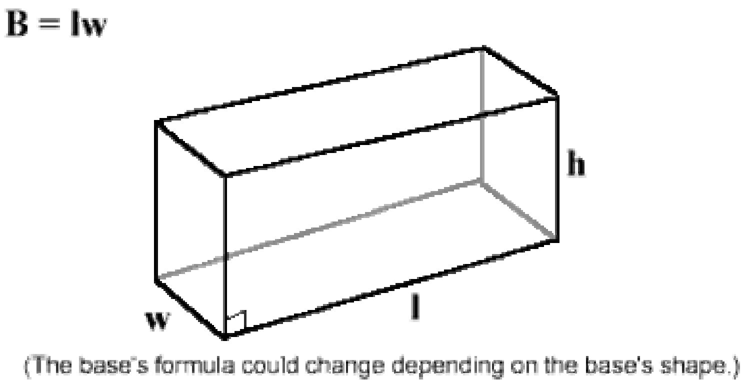


Volume of Prisms

Right Prism Volume Postulate

The volume *V* of any right prism is the product of *B*, the area of the base, and the height *h* of the prism.

Formula: $V = Bh$



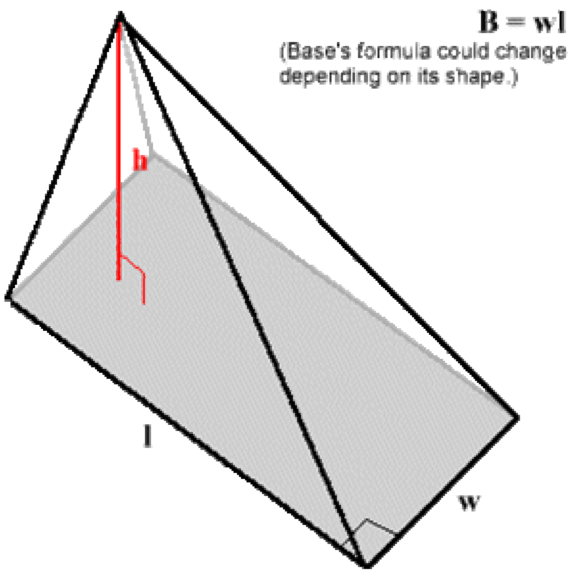
Pyramids

A *pyramid* is a polyhedron with a single base and lateral faces that are all triangular. All lateral edges of a pyramid meet at a single point, or *vertex*.

Pyramid Volume Theorem

The volume *V* of any pyramid with height *h* and a base with area *B* is equal to one-third the product of the height and the area of the base.

Formula: $V = (1/3)Bh$

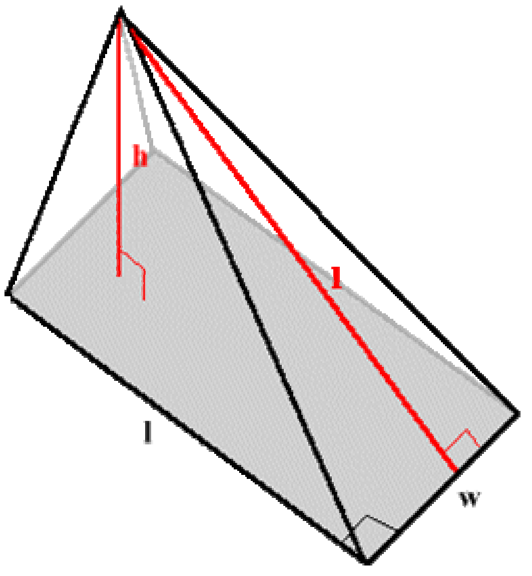


A *regular pyramid* is a pyramid that has a base that is a regular polygon and with lateral faces that are all congruent isosceles triangles.

<u>Regular Pyramid Area Theorem</u>

The area *L* of any regular pyramid with a base that has perimeter *P* and with slant height *l* is equal to one-half the product of the perimeter and the slant height.

Formula: *L = .5Pl*

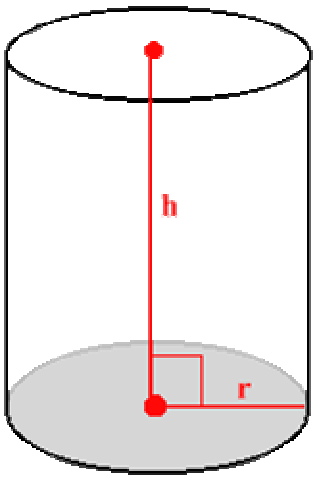


Cylinders

<u>Cylinder Volume Theorem</u>

The volume *V* of any cylinder with radius *r* and height *h* is equal to the product of the area of a base and the height.

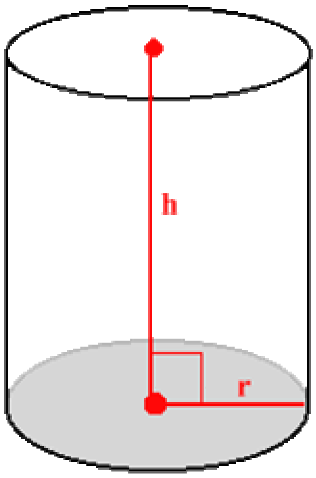
Formula: $V = (PI)r^2h$



<u>Cylinder Area Theorem</u>

For any right circular cylinder with radius *r* and height *h*, the total area *T* is two times the area of the base plus the lateral area (2(PI)rh).

Formula: $T = 2(PI)rh + 2(PI)r^2$

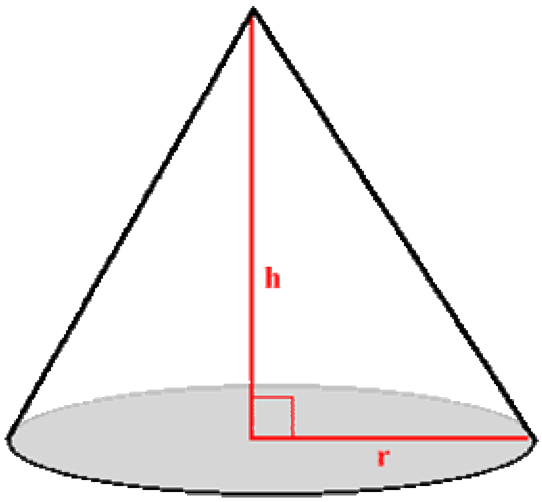


Cones

Cone Volume Theorem

The volume *V* of any cone with radius *r* and height *h* is equal to one-third the product of the height and the area of the base.

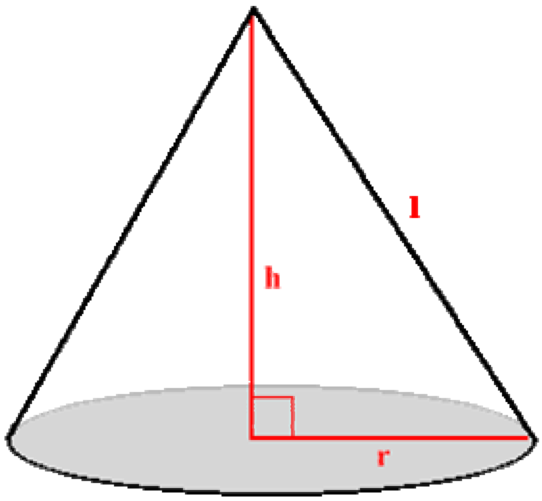
Formula: $V = (1/3)(PI)r^2h$



Cone Area Theorem

The total area *T* of a cone with radius *r* and slant height *l* is equal to the area of the base plus PI times the product of the radius and the slant height.

Formula: $T = (PI)rl + (PI)r^2$



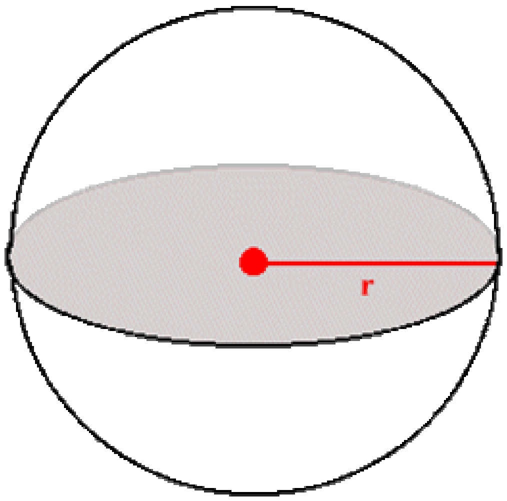
Spheres

Sphere Volume and Area Theorem

The volume *V* for any sphere with radius *r* is equal to four-thirds times the product of PI and the cube of the radius. The area *A* of any sphere with radius *r* is equal to 4(PI) times the square of the radius.

Volume Formula: $V = (4/3)(PI)r^3$

Area Formula: $A = 4(PI)r^2$



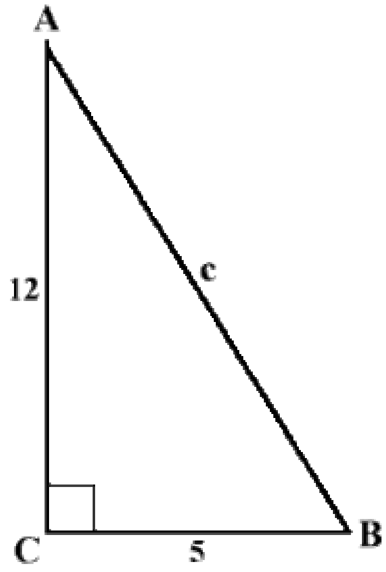
Pythagorean Theorem

One of the most famous mathematicians who has ever lived, Pythagoras, a Greek scholar who lived way back in the 6th century B.C. (back when Bob Dole was learning geometry), came up with one of the most famous theorems ever, the *Pythagorean Theorem*. It says - *in a right triangle, the square of the measure of the hypotenuse equals the sum of the squares of the measures of the two legs.* This theorem is normally represented by the following equation: $a^2 + b^2 = c^2$, where *c* represents the hypotenuse.

With this theorem, if you are given the measures of two sides of a triangle, you can easily find the measure of the other side.

Example

1. Problem: Find the value of c.



Solution: $a^2 + b^2 = c^2$ Write the Pythagorean Theorem and then plug in any given information.

$\mathbf{5}^2 + \mathbf{12}^2 = c^2$ The information that was given in the figure was plugged in.

$169 = c^2$ Solve for *c*

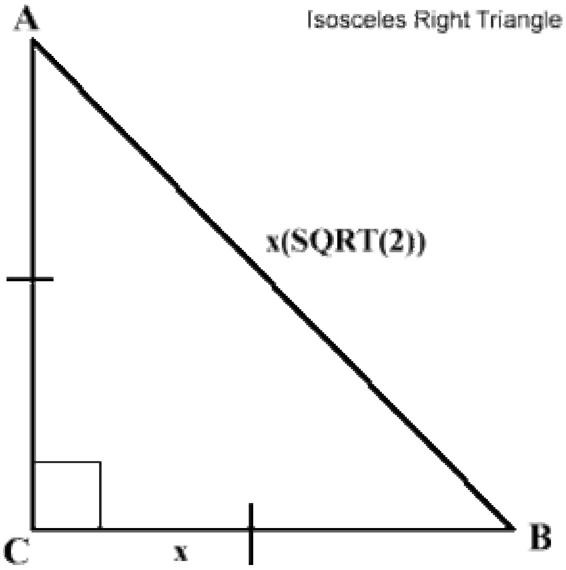
c = 13

45-45-90 Triangles

One of the special right triangles which we deal with in geometry is an isosceles right triangle. These triangles are also known as *45-45-90 triangles* (so named because of the measures of their angles). There is one theorem that applies to these triangles. It is stated below.

***In a 45-45-90 triangle, the measure of the hypotenuse is equal to the measure of a leg multiplied by SQRT(2)*.**

The following figure presents the theorem in graphical terms.

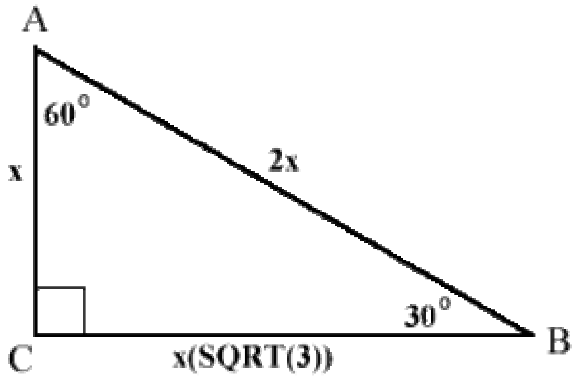


30-60-90 Triangles

There's another kind of special right triangle which we deal with all the time. These triangles are known as *30-60-90 triangles* (so named because of the measures of their angles). There is one theorem that applies to these triangles. It is stated below.

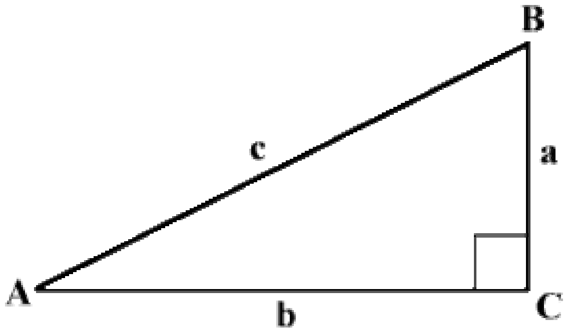
In a 30-60-90 triangle, the measure of the hypotenuse is two times that of the leg opposite the 30º angle. The measure of the other leg is SQRT(3) times that of the leg opposite the 30º angle.

The following figure presents the theorem in graphical terms.



Trigonometric Ratios

While the word *trigonometry* strikes fear into the hearts of many, we made it through (amazing as it may seem to us), and hope to help you through it, too! Each of the three basic trigonometric ratios are shown below.



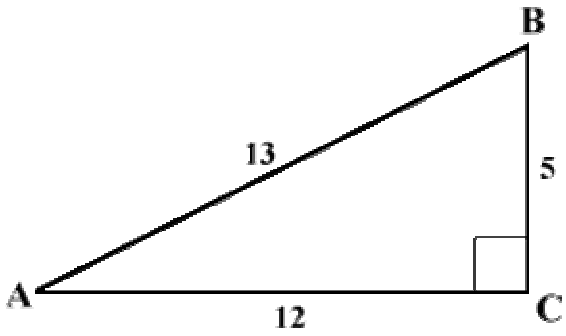
sine **of** ***angle A*** **= (measure of opposite leg)/(measure of hypotenuse). In the figure, the** ***sin*** **of** ***angle A*** **= (a/c).**

cosine **of** ***angle A*** **= (measure of adjacent leg)/(measure of hypotenuse). In the figure, the** ***cos*** **of** ***angle A*** **= (b/c).**

tangent **of** ***angle A*** **= (measure of opposite leg)/(measure of adjacent leg). In the figure, the** ***tan*** **of** ***angle A*** **= (a/b).**

Example

1. Problem: Find ***sin A, cos A,*** **and** ***tan A*****.**



Solution: sine = (opposite/hypotenuse)
sine = 5/13

cosine = (adjacent/hypotenuse)
cos = 12/13

tangent = (opposite/adjacent)
tan = 5/12

Be aware that, although the example above seems to indicate otherwise, the values for the trigonometric ratios depend on the measure of the angle, not the measures of the triangle's sides.

Story Problems

Many problems ask that you find the measure of an angle or a segment that cannot easily be measured. Problems of this kind can often be solved by the application of trigonometry. Below is an example problem of this type.

1. Problem: A ladder 12 meters long leans against a building. It rests on the wall at a point 10 meters above the ground. Find the angle the ladder makes with the ground.

Solution: Make sure you know what is being asked. Then use the given information to draw and label a figure. Here's our idea of a figure for this problem: